OCR
RECOGNISING ACHIEVEMENT

ADVANCED GCE

MATHEMATICS

Core Mathematics 4

4724

Candidates answer on the Answer Booklet

OCR Supplied Materials:
- 8 page Answer Booklet
- List of Formulae (MF1)

Other Materials Required:
None

Tuesday 13 January 2009
Morning

Duration: 1 hour 30 minutes

4724

INSTRUCTIONS TO CANDIDATES

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets **[ ]** at the end of each question or part question.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- This document consists of **4** pages. Any blank pages are indicated.

1 Simplify $\dfrac{20-5x}{6x^2-24x}$. **[3]**

2 Find $\displaystyle\int x\sec^2 x\,\mathrm{d}x$. **[4]**

3 **(i)** Expand $(1+2x)^{\frac{1}{2}}$ as a series in ascending powers of x, up to and including the term in x^3. **[3]**

(ii) Hence find the expansion of $\dfrac{(1+2x)^{\frac{1}{2}}}{(1+x)^3}$ as a series in ascending powers of x, up to and including the term in x^3. **[5]**

(iii) State the set of values of x for which the expansion in part **(ii)** is valid. **[1]**

4 Find the exact value of $\displaystyle\int_0^{\frac{1}{4}\pi}(1+\sin x)^2\,\mathrm{d}x$. **[6]**

5 **(i)** Show that the substitution $u=\sqrt{x}$ transforms $\displaystyle\int\frac{1}{x(1+\sqrt{x})}\,\mathrm{d}x$ to $\displaystyle\int\frac{2}{u(1+u)}\,\mathrm{d}u$. **[3]**

(ii) Hence find the exact value of $\displaystyle\int_1^9\frac{1}{x(1+\sqrt{x})}\,\mathrm{d}x$. **[5]**

6 A curve has parametric equations

$$x=t^2-6t+4,\quad y=t-3.$$

Find

(i) the coordinates of the point where the curve meets the x-axis, **[2]**

(ii) the equation of the curve in cartesian form, giving your answer in a simple form without brackets, **[2]**

(iii) the equation of the tangent to the curve at the point where $t=2$, giving your answer in the form $ax+by+c=0$, where a, b and c are integers. **[5]**

7 **(i)** Show that the straight line with equation $\mathbf{r}=\begin{pmatrix}2\\-3\\5\end{pmatrix}+t\begin{pmatrix}1\\4\\-2\end{pmatrix}$ meets the line passing through (9, 7, 5) and (7, 8, 2), and find the point of intersection of these lines. **[6]**

(ii) Find the acute angle between these lines. **[4]**

8 The equation of a curve is $x^3 + y^3 = 6xy$.

(i) Find $\frac{dy}{dx}$ in terms of x and y. **[4]**

(ii) Show that the point $\left(2^{\frac{4}{3}}, 2^{\frac{5}{3}}\right)$ lies on the curve and that $\frac{dy}{dx} = 0$ at this point. **[4]**

(iii) The point (a, a), where $a > 0$, lies on the curve. Find the value of a and the gradient of the curve at this point. **[4]**

9 A liquid is being heated in an oven maintained at a constant temperature of 160 °C. It may be assumed that the rate of increase of the temperature of the liquid at any particular time t minutes is proportional to $160 - \theta$, where θ °C is the temperature of the liquid at that time.

(i) Write down a differential equation connecting θ and t. **[2]**

When the liquid was placed in the oven, its temperature was 20 °C and 5 minutes later its temperature had risen to 65 °C.

(ii) Find the temperature of the liquid, correct to the nearest degree, after another 5 minutes. **[9]**

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1		Attempt to factorise numerator and denominator	M1	$\frac{A}{f(x)}+\frac{B}{g(x)}$;fg= $6x^2-24x$
		Any (part) factorisation of both num and denom	A1	Corres identity/cover-up
		Final answer = $-\frac{5}{6x}, \frac{-5}{6x}, \frac{5}{-6x}, -\frac{5}{6}x^{-1}$ Not $-\frac{\frac{5}{6}}{x}$	A1	
			3	
2		Use parts with $u=x, \mathrm{d}v=\sec^2 x$	M1	result $f(x)+/-\int g(x)\,\mathrm{d}x$
		Obtain correct result $x\tan x-\int \tan x\,\mathrm{d}x$	A1	
		$\int \tan x\,\mathrm{d}x = k\ln\sec x$ or $k\ln\cos x$, where $k=1$ or -1	B1	or $k\ln\lvert\sec x\rvert$ or $k\ln\lvert\cos x\rvert$
		Final answer = $x\tan x-\ln\lvert\sec x\rvert+c$ or $x\tan x+\ln\lvert\cos x\rvert+c$	A1	
			4	
3	(i)	$1+\frac{1}{2}.2x+\frac{\frac{1}{2}.-\frac{1}{2}}{2}\left(4x^2 \text{ or } 2x^2\right)+\frac{\frac{1}{2}.-\frac{1}{2}.-\frac{3}{2}}{6}\left(8x^3 \text{ or } 2x^3\right)$	M1	
		$= 1+x$	B1	
		$\ldots -\frac{1}{2}x^2+\frac{1}{2}x^3$ (AE fract coeffs)	A1 (**3**)	For both terms
	(ii)	$(1+x)^{-3}=1-3x+6x^2-10x^3$	B1	or $(1+x)^3=1+3x+3x^2+x^3$
		Either attempt at their **(i)** multiplied by $(1+x)^{-3}$	M1	or **(i)** long div by $(1+x)^3$
		$1-2x$ …. √ $1+(a-3)x$	A1	f.t. **(i)** = $1+ax+bx^2+cx^3$
		… + $\frac{5}{2}x^2$…. √ $(-3a+b+6)x^2$	A1	
		… $-2x^3$ √ $(6a-3b+c-10)x^3$	A1 (**5**)	(AE fract.coeffs)
	(iii)	$-\frac{1}{2}<x<\frac{1}{2}$, or $\lvert x\rvert<\frac{1}{2}$	B1 (**1**)	
			9	

4		Attempt to expand $(1+\sin x)^2$ and integrate it	*M1	Minimum of $1+\sin^2 x$
		Attempt to change $\sin^2 x$ into $f(\cos 2x)$	M1	
		Use $\sin^2 x = \frac{1}{2}(1-\cos 2x)$	A1	dep M1 + M1
		Use $\int \cos 2x \, dx = \frac{1}{2}\sin 2x$	A1	dep M1 + M1
		Use limits correctly on an attempt at integration	dep* M1	Tolerate g $(\frac{1}{4}\pi)$ − 0
		$\frac{3}{8}\pi - \sqrt{2} + \frac{7}{4}$ AE(3-term)F	A1	WW 1.51… → M1 A0
			6	

5	**(i)**	Attempt to connect du and dx, find $\frac{du}{dx}$ or $\frac{dx}{du}$	M1	But not e.g. du = dx
		Any correct relationship, however used, such as dx = 2u du	A1	or $\frac{du}{dx} = \frac{1}{2}x^{-½}$
		Subst with clear reduction (≥ 1 inter step) to **AG**	A1 (**3**)	WWW
	(ii)	Attempt partial fractions	M1	
		$\frac{2}{u} - \frac{2}{1+u}$	A1	
		√ $A \ln u + B \ln(1+u)$	√A1	Based on $\frac{A}{u} + \frac{B}{1+u}$
		Attempt integ, change limits & use on f(u)	M1	or re-subst & use 1 & 9
		$\ln \frac{9}{4}$ AEexactF (e.g. 2 ln 3 –2 ln 4 + 2 ln 2)	A1 (**5**)	Not involving ln 1
			8	

6	(i)	Solve $0 = t - 3$ & subst into $x = t^2 - 6t + 4$	M1	
		Obtain $x = -5$	A1 (2)	(–5,0) need not be quoted
		N.B. If **(ii)** completed first, subst $y = 0$ into their cartesian eqn (M1) & find x (no f.t.) (A1)		
	(ii)	Attempt to eliminate t	M1	
		Simplify to $x = y^2 - 5$ ISW	A1 (2)	
	(iii)	Attempt to find $\frac{dy}{dx}$ or $\frac{dx}{dy}$ from cartes or para form	M1	Award anywhere in Que
		Obtain $\frac{dy}{dx} = \frac{1}{2t-6}$ or $\frac{1}{2y}$ or (-)$\frac{1}{2}(x+5)^{-\frac{1}{2}}$	A1	
		If $t = 2$, $x = -4$ and $y = -1$	B1	Awarded anywhere in **(iii)**
		Using their num (x, y) & their num $\frac{dy}{dx}$, find tgt eqn	M1	
		$x + 2y + 6 = 0$ AEF(without fractions) ISW	A1 (5)	
			9	
7	(i)	Attempt direction vector between the 2 given points	M1	
		State eqn of line using format (**r**) = (either end) + s(dir vec)	M1	's' can be 't'
		Produce 2/3 eqns containing t and s	M1	2 different parameters
		Solve giving $t = 3$, $s = -2$ or 2 or -1 or 1	A1	
		Show consistency	B1	
		Point of intersection = $(5,9,-1)$	A1 (6)	
	(ii)	Correct method for scalar product of 'any' 2 vectors	M1	Vectors from this question
		Correct method for magnitude of 'any' vector	M1	Vector from this question
		Use $\cos\theta = \frac{\mathbf{a.b}}{\lvert\mathbf{a}\rvert\lvert\mathbf{b}\rvert}$ for the correct 2 vectors $\begin{pmatrix}1\\4\\-2\end{pmatrix}$ & $\begin{pmatrix}2\\-1\\3\end{pmatrix}$	M1	Vects may be mults of dvs
		62.2 (62.188157…) 1.09 (1.0853881)	A1 (4)	
			10	

8	**(i)**	$\frac{d}{dx}\left(y^3\right)=3y^2\frac{dy}{dx}$	B1	
		Consider $\frac{d}{dx}(xy)$ as a product	M1	
		$= \quad x\frac{dy}{dx}+y$	A1	Tolerate omission of '6'
		$\frac{dy}{dx}=\frac{6y-3x^2}{3y^2-6x}$ ISW AEF	A1 (**4**)	
	(ii)	$x^3=2^4$ or 16 and $y^3=2^5$ or 32	*B1	
		Satisfactory conclusion	dep* B1	**AG**
		Substitute $\left(2^{\frac{4}{3}},2^{\frac{5}{3}}\right)$ into their $\frac{dy}{dx}$	M1	or the numerator of $\frac{dy}{dx}$
		Show or use calc to demo that num = 0, ignore denom **AG**	A1 (**4**)	
	(iii)	Substitute (a,a) into eqn of curve	M1	& attempt to state 'a = …'
		$a=3$ only with clear ref to $a\neq 0$	A1	
		Substitute $(3,3)$ or (their a, their a) into their $\frac{dy}{dx}$	M1	
		-1 only WWW	A1 (**4**)	from (their a,their a)
			12	
9	**(i)**	$\frac{d\theta}{dt}=\ldots$	B1	
		$k(160-\theta)$	B1 (**2**)	The 2 @ 'B1' are indep
	(ii)	Separate variables with $(160-\theta)$ in denom; or invert	*M1	$\int\frac{1}{160-\theta}d\theta=\int k,\frac{1}{k},1\,dt$
		Indication that LHS = $\ln f(\theta)$	A1	If wrong ln, final 3@A = 0
		RHS = kt or $\frac{1}{k}t$ or t $(+c)$	A1	
		Subst. $t=0,\theta=20$ into equation containing 'c'	dep* M1	
		Subst $t=5,\theta=65$ into equation containing 'c' & 'k'	dep* M1	
		$c=-\ln 140$ (-4.94) ISW	A1	
		$k=\frac{1}{5}\ln\frac{140}{95}$ (≈ 0.077 or 0.078) ISW	A1	
		Using their 'c' & 'k', subst t = 10 & evaluate θ	dep*M1	
		$\theta=96(95.535714)$ $\left(95\frac{15}{28}\right)$	A1 (**9**)	
			11	